AN ELECTRODYNAMOMETER USING THE VIBRATION TELESCOPE

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1. Introductory.—The employment of a telescope with a vibrating objective did good service as an aid to the interferometry of vibrating systems. It seemed worth while therefore to see what could be got out of it, when used in connection with a telephone only, as a dynamometer. The experiments are of interest both because of the vibratory phenomena observable and in view of the peculiar method of optic observation developed. Its possible use for finding the magnetic field within a helix of unknown constants deserves mention.

2. Apparatus.—A front view (elevation) of the design is given in figure 1 and an enlarged detail (side view) in figure 2. The apparatus consists of a rigid rectangular frame-work of $\frac{1}{4}$ inch gas pipe, A, B, C, D, EE', F being the foot attached to a tripod. There may be a steadying foot at C'. A and D are attached to EE' by the stout clamps c, c'', so that EE' lying behind the plane of $ABCD$, admits of the attachment of a suitable clamp c', by which the telephone ih may be held in the same plane. B and C may be forced apart slightly by the screw n controlled by the broad thumb nut m, the conical end of n rotating in a socket of the cap p.

The vibrating system consists of the bifilar wires of phosphor bronze dd' and the frame of the lens f which is the movable objective of the telescope T, the latter part containing the ocular and a plate micrometer (cm. divided in 100 parts). T may be at a considerable distance (50 cm. or more) from f, and supported by a convenient standard. The frame of the lens (which must hold it securely, cement being used if necessary) is of light sheet metal, the parts gg' being of sheet iron (about 0.05 cm. thick) so as to be attracted by the magnet, i, of the telephone. The stiff cross wires, r,s, of the frame are either soldered to the bifilar system dd' or otherwise attached to it (soft sealing wax does very well for temporary experimental purposes).

The attachment and tension-control of the bifilar suspension is finally to be described, as its period must be synchronized with the alternating current. Results are obtainable only when the two periods are strictly in unison. In figure 1 the wires dd' are looped around a groove in the pipe D below, and the upper ends of dd' after passing a similar groove in A are bent around the post a, a', and wound respectively around the snugly fitting screws, b, b', the ends being secured against sliding by a fine hack saw cut in the screws. To stretch a wire it is passed from the notch in b once or twice around it, thence around a, downward by the groove to D and then up in the corresponding way to b'.

The lens carriage gg' is then attached with cement (after the wires are evenly stretched) with the crosswires r, s on the opposite side of dd' to the pull of the magnet i. The magnet, in addition to the cement, thus guards against slipping. On turning the screws b and b' any degree of tension may be imparted to the wires bb', roughly. This simple device worked surprisingly well, and wires of different kinds may be easily inserted or replaced, the lens system being subsequently attached with cement; but it is better to loop the lower part of dd' around a special roller G, as indicated in figure 2, and used in my later tests, with the object of more easily reaching an equality of tension in the wires d and d'. The tensions are then roughly changed by the screw and nut u.

The approximate tension having thus been obtained by the screws u, b, b', the finer variations are imparted by the screw mn which flexes the elastic rectangle $ABCDE$ and thus gives to the bifilars d, d' exactly the tension required. It is at the thumb nut m that all adjustment is made during observation.

In my apparatus the rectangle was about 50 cm. long and 12 cm. wide. The wires d, d' about 1.5 cm. apart and each about 0.025 mm. in diameter. Wires as thick as this require sharp adjustment as to tension, but the method given proved quite satisfactory particularly as it is little disturbed by manipulation. The tension is sufficient to admit of an air gap of less than a millimeter between the plate g' and the magnet i of the telephone. Later the telephone was also put on a spring-micrometer screw for fine adjustment.

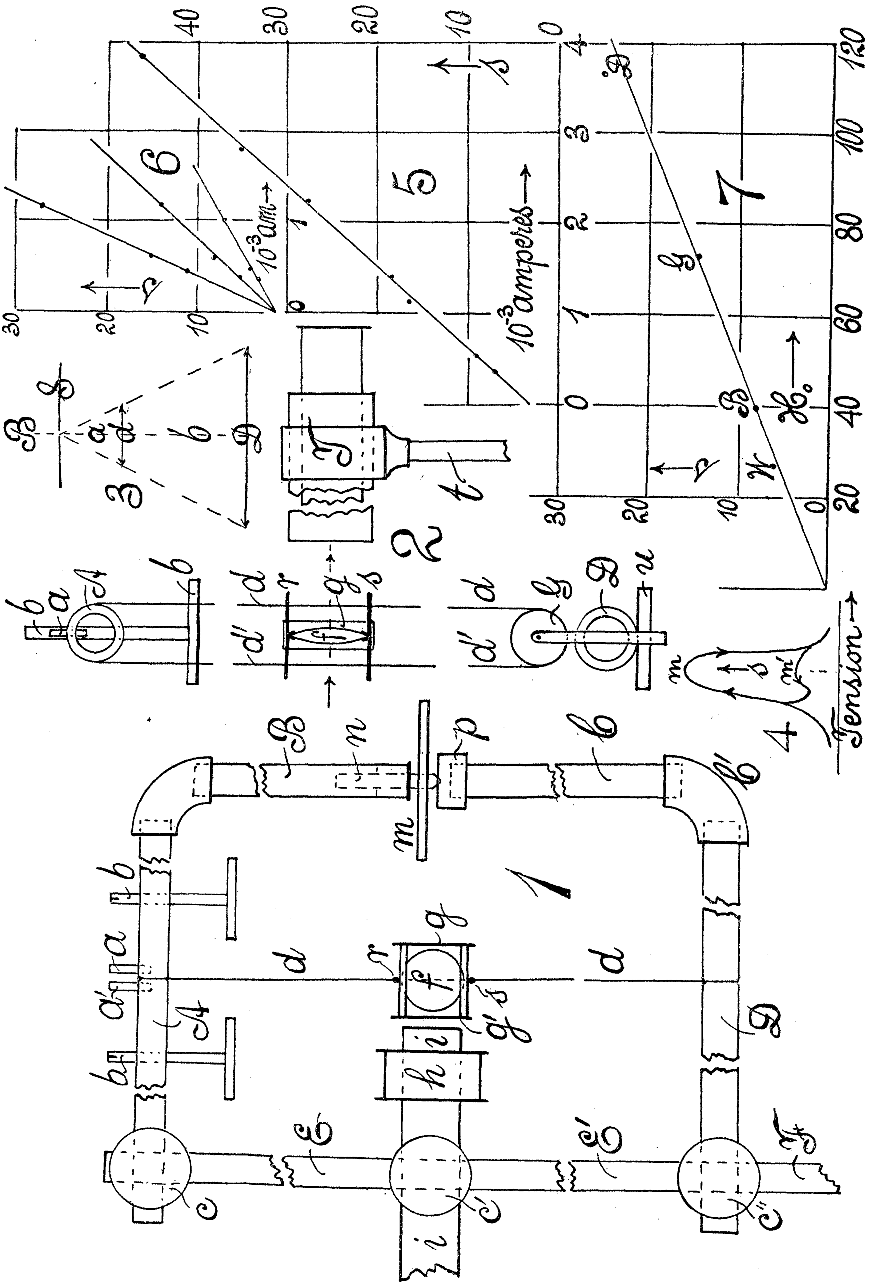
In the case of parallel rays, the displacement of the image in the ocular would be no larger than the displacement of the objective, f, figure 1. To obtain increased displacement, the method of figure 3 is available where S is the fine slit in front of a Welsbach burner at B. At d is the principal plane of the vibrating objective and at D the micrometer plate in the ocular. Again, if the length d represents the double amplitude of the objective and the sides of the triangle be drawn from S, the intercept D will represent the displacement in the ocular. If the distance Sd be a and dD, b, we may write

$$1/a + 1/b = 1/f \tag{1}$$

where f is the principal focal distance of the objective. Hence

$$D/d = b/f \tag{2}$$

Theoretically, therefore, any degree of magnification is possible by increasing b (the distance of T and f, fig. 2) and decreasing f. In the former case, some means of controlling the thumb nut m, figure 1, from a distance would have to be provided. In the latter the lens f would have to be achromatic. In the present experiments I first used an ordinary spectacle lens at f with a slotted screen between it and the slit to diminish chromatic aberration; but there is no objection to the use of an achromatic lens at f, as was done later, particularly since a breadth of a few millimeters of lens will suffice, for there is an abundance of light.


10⁻³ amperes →
10⁻³ am →
Tension →
1
2
3
4
5
6
7

To measure the width of the image band of light produced by the vibration, the ocular T should be on an axle t with slight friction. In my earlier improvised apparatus, single scale-parts (0.1 mm.) only could be guaranteed; but with a perfected optical system there is no reason why tenths of scale-parts should not be equally trustworthy. This makes a scale of 1000 parts in the ocular.

3. *Observations.*—As a source or alternating current I selected a small induction coil with a mercury break (Kohlrausch's design) to facilitate the initial tests. This was put in series with a rheostat (to 30,000 ohms), a Siemens's precision dynamometer reading to within milliamperes, an ordinary telephone to indicate the continuous action of the coil and the vibrator above described.

The Siemens's dynamometer was first standardized with a Clarke cell. Accepting the effective current i as being

$$i = C\sqrt{\varphi}$$

where φ is the deflection on a scale at about 1 meter of distance, the constant $C = 1.12 \times 10^{-3}$ relative to amperes was found and the mean resistance of the coils included about 250 ohms. Virtual currents of 10^{-4} amperes would escape detection.

The coil was now started and measurements made simultaneously both at the Siemens's dynamometer and at the vibrating telephone (slit distance a = 35 cm., ocular distance b = 75 cm.), with good results, the virtual currents of the dynamometer being compared with the width of the image band (in scale-parts) seen in the vibration telescope. To obtain different virtual currents, resistances from 10,000 to 2000 ohms were put into the circuit. The dynamometer showed deflections from 2 to 20 cm. on the scale. For larger deflections the coil current would have been too irregular for use.

The vibrator indicated about 10 scale-parts per milliampere, and readings even beyond 5 or 10 milliamperes would be possible. The deflections were fairly proportional to the current, indicating that with a good optical system virtual currents as small as 10^{-5} ampere should have been perceptible. The apparatus is thus very well worth further development and would be particularly useful where alternators with a definite period are in question.

The method of observation consists in gradually increasing the tension of the wire from a slightly low value, to beyond the maximum tension. In this case the band widens from a relatively small width to the maximum and then abruptly falls off to a small value. To repeat the observation the tension must often again be reduced to the low value and the whole operation repeated; but after some practice maximum may be reached in the reverse direction unless the band has become too narrow.

When the current is broken and thereafter closed, a low width of band is obtained which will not widen unless the operation described is carried out from low tension. In other words there are often two cases of equilibrium for each

current, corresponding to very different band widths. This is a curious result, for it means, virtually, that the magnetic forces and the stresses are in the relation of a doubly inflected curve to each other, so that there are three intersections, two for stable vibratory equilibrium; or else the two harmonic systems, the electrical and the mechanical, may vibrate in the same or in opposed phases. The cycle in figure 4 indicates the general relation of the band width s to the tensions, there being two maxima at m and m'.

Similarly each current requires its own particular maximum tension, which increases with the current. The difference is not large, but effective and for this reason the fine tension adjustment is essential.

4. *Further observations.*—The apparatus was then improved in a variety of ways, chiefly by the insertion of a small vibration objective, about 1 cm. in diameter, achromatic and with a focal distance of but 5.8 cm. In this case the distance a and b, figure 3, could be decreased to 7 cm. and 35 cm. and the observer was thus conveniently near the adjusting screw. The slit image was white and about a scale-part in width. There would have been no difficulty in using much greater magnification.

An example of the results (band-width s in scale parts) is given in figure 5. The constant of the dynamometer was now $C = 10^{-3} \times 0.87$ relative to amperes and the total resistance in circuit 710 ohms. The frequency, as before, was estimated at about 15 per second.

The results were a considerable improvement on the preceding and the discrepancies as a rule lie within 5×10^{-5} ampere. They are much more liable to be in the dynamometer than in the vibrator, as the former was not well adapted for these small currents. The deflections begin with 3 scale-parts (initial slit breadth) and not at zero; but as this appears merely as an initial constant, it is not of consequence.

If we compute the coefficient of induction as

$$L^2\omega^2 = \Delta(i^2(R + r)^2)/\Delta i^2$$

(Δ being a differential symbol) from the first and fifth, second and sixth, etc., observations, data for $L\omega$ and L follow.

5. *Effect of frequency.*—A special mercury interruptor was now made having as its distinctive feature contrivances by which the mercury surface was washed and thus could always be kept clean and bright. It was furthermore adapted to give different frequencies. The apparatus functioned admirably for days, frequent washing presupposed. Different frequencies, obtained by a sliding weight, were estimated from the moments of inertia as $n = 10$, 15, and 20. The latter could just be counted in groups of 4 vibrations with a stop-watch. Higher frequencies were obtainable by using stiffer springs.

An example of the results obtained with this apparatus is given in figure 6, for the phosphor bronze bifilar differently stretched. All gave evidence of the peculiar fact that the sensitiveness increases in marked degree with the frequency.

It is difficult to account for this effect of frequency, so peculiarly marked in the instance given, where the observations were good. If different harmonics are in action, the overtones would have to respond in case of the wires under less tension, and this seems unlikely. The behavior of very tense steel wires, moveover, was the reverse of this. It is probable that the phenomena of §6 contain the clue.

6. *Adjustable telephone.*—The question as to the most advantageous position of the telephone is thus variously important. Consequently the telephone *ih*, figure 1, was mounted on a stout flat steel spring controlled by a micrometer screw. By actuating this, the face near g' could be approached as near g' as permissible, or withdrawn to a remoter position, with precision. The experiments made at length showed that within a wide range of tensions and of telephone positions, a particular degree of approach of the telephone corresponded to each particular stress of wire. Unless these paired positions are selected, the bifilar system does not respond. Tense wires require a nearer telephone; less tense wires a more remote telephone (even 0.5 cm.), within wide limits. Therefore the condition of resonance may be reached either by adjusting the telephone on the micrometer screw for a given tension of wire; or by changing the tension of the wires for a fixed telephone. Between the admissible limits of tension, the sensitiveness passes through a flat maximum.

With a distance of not more than 50 cm. between slit and ocular, a judicious disposition of parts eventually gave me 40 ocular scale parts per virtual milliampere, viz.,

n	$i \times 10^3$	$s \times 10^2$
	amperes	*cm.*
15	0.67	27
20	0.67	27

so that here the above effect of frequency, n, is no longer apparent.

7. *Coil tester.*—An interesting application of the above apparatus, where a definite frequency is usually assigned, is its possible use for measuring the magnetic fields of different coils. For this purpose I wound a long thin primary of an induction coil, which when inserted into the coil to be tested, should, from the measurement of the current induced in the secondary in question and in the absence of other mutual inductions, give the constants of the secondary. As many of the coils to be tested were of small internal diameter, the primary was wound on a long iron tube, fine wire being necessary. The data are for the

Iron tube

Diameter outside........0.635 cm.
Length ....55 cm.
Walls.............0.08 cm. thick

Helix

Diameter outside..........0.7 cm.
Wire........0.034 cm. in diameter
n_1/l_1 ......21 turns per linear cm.

If L is the coefficient of induction per turn of primary, the total induction is

$$B = LN_1 i \tag{1}$$

Hence the electromotive force induced in the secondary becomes

$$e = LN_1N_2(di_1/dt) \tag{2}$$

If the field of the secondary per unit current is put

$$H_2 = 4\pi N_2/l_2 \tag{3}$$

for N_2/l_2 turns per linear centimeter, and r the resistance of this coil and its circuit, we may compare any two coils by the equation

$$\frac{e_2}{e'_2} = \frac{N_2}{N'_2} = \frac{H_2 l_2}{H'_2 l'_2} = \frac{i_2 r_2}{i'_2 r'_2} \tag{4}$$

e_2 and e'_2 being the electromotive forces, and i_2 and i'_2 the currents induced in the two secondaries in question. Thus

$$\frac{H_2}{H'_2} = \frac{i_2 r_2/l_2}{i'_2 r'_2/l'_2} = \frac{s_2/l_2}{s'_2/l'_2} \tag{5}$$

it being assumed that the resistance r in the secondary circuit is made so large that the inductive resistance vanishes.

The coil tester was now thrust through a variety of helices, differing in shape and construction.

Figure 7 gives the results read off for these coils at the vibrator (s), when a high resistance was added to the circuit and Ri constant. The relation of H_0 and s as seen in figure 7, is linear for coils W, B, G, which were about of the same length and wound on non-conductors or split brass. The result for coil D, which was about 1.8 times longer is low, probably owing to mutual induction, as this coil was wound on a thick brass tube without fissure. Similarly it made very little difference whether the two helices wound side by side throughout G, were used in parallel or in series. Finally a number of single layer coils of lengths 10, 20, 30, 40 cm. were wound on stout glass tubing and compared with B. Provided care was taken to prevent the induced currents from heating the telephone, the band width s increased throughout proportionally to the number of turns of wire in the secondary and under good conditions at a rate of about $s = 0.06$ scale parts per turn of wire.